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Structures in the Gauge/Gravity Duality Cascade

Girma Hailu*and S.-H. Henry Tye†

Newman Laboratory for Elementary Particle Physics
Cornell University
Ithaca, NY 14853

Abstract:
We study corrections to the anomalous mass dimension and their effects in the Seiberg duality cascade in the Klebanov-Strassler throat, where $\mathcal{N} = 1$ supersymmetric $SU(N+M) \times SU(N)$ gauge theory with bifundamental chiral superfields and a quartic tree level superpotential in four dimensions is dual to type IIB string theory on $AdS_5 \times T^{1,1}$ background. Analyzing the renormalization group flow of the couplings on the gauge theory side, we calculate corrections to the anomalous mass dimension. Applying gauge/gravity duality, we then show that the corrections reveal structures on the supergravity side with steps appearing in the running of the fluxes and the metric. The "charges" at the steps provide a gravitational source for Seiberg duality transformations. The magnitudes of these charges confirm that the theory flows to a baryonic branch rather than to a confining branch. The cosmological implication of the duality cascade and the gauge/gravity duality on the brane inflationary scenario and the cosmic microwave background radiation is pointed out.

*hailu@lepp.cornell.edu
†tye@lepp.cornell.edu

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1. Introduction

Recent developments in flux compactification in string theory have provided us with many explicit realizations of the brane world scenario with stabilized moduli [1, 2]. In a typical solution in type IIB theory, the compactified manifold has a number of warped throats. It is likely that our standard model particles are open string modes of a stack of D-branes sitting at the bottom of such a throat. A prime example of such a throat is the Klebanov-Strassler (KS) throat [3]; i.e., a warped deformed conifold in type IIB theory on $AdS_5 \times T^{1,1}$ background. Its gauge theory dual is $\mathcal{N} = 1$ supersymmetric $SU(N+M) \times SU(N)$ with bifundamental chiral superfields and a quartic tree level superpotential which undergoes a cascade of Seiberg duality transformations [8]. Here, we like to explore the properties of such a throat in some details. We will start with studying the running of the couplings in the gauge theory and calculate corrections to the anomalous mass dimension which dictate the flow of the gauge theory. The corrections depend on the ranks of the gauge groups in the duality cascade. We then apply gauge/gravity duality and find that including the corrections to the anomalous dimension on the gauge theory side reveals structures with steps in the metric and the fluxes on the gravity side.

These steps may be observable in cosmology. This point is perhaps best expressed by quoting WMAP [4] : "a very small fractional change in the inflaton potential amplitude, $c \sim 0.1\%$, is sufficient to cause sharp features in the angular power spectrum." Sharp features and/or non-Gaussianity in the cosmic microwave background radiation due to steps in the potential have been studied [5]. The possibility of detecting and measuring the duality cascade is a strong enough motivation to study the throat more carefully. Although both Seiberg duality and gauge/gravity duality are strongly believed to be true, neither has been proven; so a cosmological test is highly desirable. It has become clear that the brane inflationary scenario in string theory is quite robust [6]. Here, the inflaton is simply the position of the $D3$-brane. The duality cascade feature shows up in the warped geometry and in the running of the coupling (the dilaton value) in the $D3$-anti-$D3$-brane potential as steps, so the step function behavior of the throat, though small, can have distinct observable signatures in the cosmic microwave background radiation. As the $D3$-brane moves towards the bottom of the throat, the warp factor may come into play when the brane is moving ultra-relativistically (but very slowly) down a throat [7]. In this case, this cascade feature in the warped geometry appears in the Dirac-Born-Infeld action.

To find the step structure on the gravity side, we shall use a gauge/gravity duality property; i.e, each quantum gravity theory has a dual description in a (nonperturbative) gauge theory. On the gauge theory side, as the theory flows towards the infrared (IR), the larger of the two gauge factors undergoes a Seiberg duality transition as it becomes strongly coupled while the weaker factor is treated as a flavor symmetry : $SU(N+M)$ with $2N$ flavors $\rightarrow SU(2N-(N+M)) = SU(N-M)$ with $2N$ flavors [8]. Repeating such transformation, the $SU(N+M) \times SU(N)$ gauge theory undergoes a series of Seiberg duality as it flows towards the IR, i.e., the bottom of the throat. This is the duality cascade. At the l^{th} step ($l = 1, 2, ...$), the gauge theory makes the transition from the l^{th} region; i.e., $SU(N + M - (l-1)M) \times SU(N - (l-1)M)$ to the $(l+1)^{th}$ region with $SU(N+M-lM) \times SU(N-lM)$ gauge group. This duality cascade should lead to steps in the fluxes and the metric in the gravity side. To see this, let us first look at the value of the anomalous mass dimension γ, since the renormalization group flow of the couplings depends crucially on it. The $M = 0$ case is the conformal Klebanov-Witten (KW) model [12], where $\gamma_0 = -1/2$. Turning on M breaks the conformal symmetry and so should lead to a correction to γ. Intuitively, when $N >> M$, this correction is expected to be small and so is usually neglected. Here we find interesting physics associated with this correction to the anomalous dimension. Furthermore, this correction becomes substantial as we approach the IR limit.

Since the gauge theory has the obvious symmetry $M \rightarrow -M$, $N \rightarrow N + M$, γ must be even under this symmetry and so its leading order correction must have the form $M^2/N(N+M)$. On the other hand, if we turn off the superpotential, the two gauge couplings will have individual fixed points (when the other group is weakly coupled and is treated as a flavor symmetry) provided $\gamma_{N+M} = -1/2 - 3M/2N$ and $\gamma_N = -1/2+3M/2(N+M)$, respectively. So, when the superpotential is turned back on, we expect the common γ to be somewhere in between, and the renormalization group flow of the couplings take place somewhere in between too. Together with the above symmetry, we propose that γ should be the average of γ_{N+M} and γ_N. Similarly, after l steps in the Seiberg duality cascade, the gauge theory becomes $SU(N + M - lM) \times SU(N - lM)$, with the corresponding anomalous dimension,

$$\gamma = -\frac{1}{2} - \frac{3M^2}{4N(N+M)} \to -\frac{1}{2} - \frac{3M^2}{4(N-lM)(N+M-lM)} \tag{1.1}$$

Thus γ jumps from one value to another value as the renormalization group flow goes through a Seiberg duality transition. It is this jump in γ which causes the steps in the metric and in the fluxes on the gravity side. Note that, towards the bottom of the throat, $l \to N/M$, the correction in γ is no longer negligible. For the anomalous dimension to stay finite in the $N = KM$ case, $l = 1, 2, ..., K - 1$. That is, there are only $K - 1$ steps in the Seiberg duality cascade, and the infrared flow from $SU(3M) \times SU(2M)$ to $SU(2M) \times SU(M)$ takes $\gamma = -7/8$. This confirms that the theory flows to a baryonic branch rather than to a confining branch.

That the Seiberg duality cascade should introduce steps is not surprising. In the absence of the corrections to the anomalous mass dimension (1.1), the Seiberg duality cascade is completely smooth when one looks at the geometry. On the gauge theory side, we see that the gauge coupling in the renormalization flow goes from zero to infinity between duality transformations. Actual dynamics suggests that the gauge couplings should flow from weak to strong, i.e., from a small value to a large value, but never reach zero or infinity. The introduction of the anomalous dimension (1.1) prevents the flow of the gauge coupling to either zero or infinity. It also introduces the step-like behavior in the metric and the duality cascade is seen on the gravity side as well.

The rest of this paper is organized as follows. First we will give a brief background review. We then give the details on the determination of the anomalous dimension and its implications on the renormalization group flow. A study of the implication in the gravity side is done on the setting of a singular conifold. The singular conifold geometry is a good approximation to the deformed geometry in the UV region near the edge of the throat. This is good enough for our purpose here as it captures the important features of the physics: the steps with their magnitudes and radial locations. We will see that the dilaton and the 2-form NS-NS potential run with kinks and the NS-NS flux has steps. We will then continue with analyzing the supergravity side using $SU(3)$ structures and see how the corrections on the gauge theory side could give rise to geometric obstructions on the supergravity side which provide special locations and sources for Seiberg duality transformations. A full supersymmetric solution on the supergravity side containing the corrections will involve a detailed analysis of the supergravity equations of motion and their solutions and we will not attempt to do that here. We will conclude with some remarks.

2. Brief review

Klebanov and Witten found, shortly after the first example of a dual gauge/gravity theory was given by Maldacena [9], Gubser, Klebanov and Polyakov [10], and Witten [11], that type IIB string theory with a stack of N $D3$-branes on $AdS_5 \times T^{1,1}$ was dual to $\mathcal{N}=1$ supersymmetric $SU(N)\times SU(N)$ conformal gauge theory with bifundamental chiral superfields A_1 and A_2 transforming as $(\Box, \bar{\Box})$ and B_1 and B_2 transforming as $(\bar{\Box}, \Box)$ and a quartic tree level superpotential [12]. The gauge and flavor invariant quartic tree level superpotential in this theory is given by

$$W_{\text{tree}} = w\Big((A_1B_1)(A_2B_2) - (A_1B_2)(A_2B_1)\Big), \tag{2.1}$$

where color indices from the same gauge group are contracted in the gauge invariant fields (A_iB_j) and w is the tree level coupling. Let us define the classical dimensionless coupling related to the tree level coupling w by $\eta = w/\mu$, where μ has the dimension of mass. The physical β functions of the two gauge couplings are $\beta_g = 3N - 2N(1-\gamma_g)$, and that of η is given by $\beta_\eta = 1 + 2\gamma_\eta$, where the γs are the anomalous mass dimensions. Although the superpotential breaks the flavor symmetry to its diagonal version, there is enough symmetry left so that there is a common anomalous mass dimension, that is, $\gamma = \gamma_\eta = \gamma_g$. The theory has a nontrivial conformal fixed point, where the physical β functions of the two gauge couplings associated to the two group factors in $SU(N)\times SU(N)$ and that of η all vanish. This happens for the same value of the anomalous mass dimension, namely $\gamma_0 = -1/2$. So the theory is conformal with this value of γ, which is independent of N. The stack of $D3$-branes induces a 5-form R-R flux and the supergravity geometry is a warped pure $AdS_5 \times T^{1,1}$.

In the KS construction, in a series of papers by Klebanov and collaborators (with Gubser [13], with Nekrasov [14], with Tseytlin [15] and with Strassler [3]), an additional M number of $D5$-branes are wrapped near the tip over the S^2 cycle of $T^{1,1}$. These wrapped $D5$-branes become fractional $D3$-branes localized at the apex of the conifold. This enhances the gauge theory to $SU(N+M)\times SU(N)$ with A_1, $A_2 \sim (\Box, \bar{\Box})$ and B_1, $B_2 \sim (\bar{\Box}, \Box)$. This theory with the quartic tree level superpotential given by (2.1) is dual to type IIB string theory on a warped deformed conifold, with $AdS_5 \times T^{1,1}$ background. In this case, there is no value of common anomalous dimension that makes the physical β functions of the couplings vanish simultaneously; that is, the addition of the fractional branes makes the theory nonconformal. The fractional branes induce 3-form R-R flux through the S^3 cycle of $T^{1,1}$. This flux, considered as a perturbation of the $AdS_5 \times T^{1,1}$ background, induces a 2-form backreaction potential which varies with the radius of the $T^{1,1}$ and produces a logarithmic flow. It was argued that this theory undergoes a cascade of Seiberg duality transformations in the strongly coupled region with the duality transformation alternating between the two gauge group factors. The flow of the couplings would continue until, in the case where N is integral multiple of M, $SU(2M) \times SU(M)$ is left. At this point two different possible routes are discussed in the literature. In one case, a pure $SU(M)$ gauge theory is left in the infrared which undergoes confinement via gaugino condensation [3]. On the string theory side, the confinement corresponds to a deformation of the tip of the cone via geometric transition whereby the S^2 cycle is blown-down and the S^3 is blown-up with 3-form R-R flux through it. There is also a second possible route for the cascade ending in a baryonic branch of $SU(2M) \times SU(M)$ with a quantum deformed moduli space and a massless axionic moduli field [16]. Our work here confirms that the second route is the preferred one.

Assuming that the duality cascade picture is a correct description of the gauge theory, we start with determining the corrections to the anomalous dimension. Once we find the corrections to the anomalous dimension, we will study the effects on the supergravity side on the setting of the singular conifold. The singular conifold geometry is a good approximate description in the UV region near the edge of the throat which is enough for our purpose of finding the steps and the corresponding sizes. We find that the leading corrections in M/N come at orders expected from flux backreaction estimates. For instance, the leading order correction to the anomalous mass dimension comes at $\mathcal{O}(M^2/N^2)$, the gauge coupling β functions receive leading $\mathcal{O}(M^2/N)$ corrections and the dilaton runs at $\mathcal{O}(M^2/N)$. This is consistent with dual supergravity flux backreaction estimates [14, 15] where the leading order corrections to the anomalous dimension is expected to come at most at $\mathcal{O}(M^2/N^2)$. The magnitude of the corrections changes after each duality transformation as the matter content of the theory changes and this introduces steps in the backreaction H_3 flux and in the warp factor. In $e^{-\Phi}$ for the dilaton and in the B_2 NS-NS potential, it is the slope in the logarithmic running which changes at the cascade steps. The corrections grow as the cascade proceeds and the difference in the ranks of the gauge groups gets bigger. Our premise of a changing anomalous dimension as the duality cascade proceeds and the matter content of the theory changes is consistent with the picture of the theory flowing to the baryonic branch with $SU(2M) \times SU(M)$. Here the reason for the flow to a baryonic branch is because an additional Seiberg duality transformation would require an infinite "charge" at the step.

3. Seiberg duality cascade

Let us consider the $\mathcal{N} = 1$ supersymmetric $SU(N+M) \times SU(N)$ gauge theory with chiral superfields transforming as $A_1, A_2 \sim (\square, \bar{\square})$ and $B_1, B_2 \sim (\bar{\square}, \square)$ in the KS construction. The quantity $\gamma = \gamma_A + \gamma_B$ stands for the anomalous dimension of any one of the gauge invariant objects made out of the bifundamental chiral superfields, which contains one A and one B superfields and which must have the same anomalous dimension because of $SU(2)$ global flavor symmetry in the theory. Let us denote the gauge coupling of the larger group, which is $SU(N+M)$ in the 1^{st} region, by g_1, and that of the smaller group, which is $SU(N)$ in the 1^{st} region, by g_2, and define $T_1 \equiv -2\pi i\tau_1 = 8\pi^2/g_1^2$ and $T_2 \equiv -2\pi i\tau_2 = 8\pi^2/g_2^2$. Suppose we start with taking one gauge group as a weakly coupled gauge theory relative to the other, then we can treat that weaker group as a flavor symmetry. The running of the physical couplings [17] with appropriate normalization of the gauge chiral superfields can then be written as

$$\beta_1 = \mu\frac{dT_1(1)}{d\mu} = 3(N+M) - 2N(1-\gamma_1(1)), \tag{3.1}$$

$$\beta_2 = \mu\frac{dT_2(1)}{d\mu} = 3N - 2(N+M)(1-\gamma_2(1)), \tag{3.2}$$

and

$$\beta_\eta = \mu\frac{d\eta(1)}{d\mu} = 1 + 2\gamma_\eta(1), \tag{3.3}$$

where we have not yet identified the γs. We have put different indices on $\gamma_1(l)$ in (3.1), on $\gamma_2(l)$ in (3.2) and on $\gamma_\eta(l)$ in (3.3) since the two gauge groups have different ranks and "see" different numbers of flavors and would tend to flow with different anomalous dimensions. The number "1" in the parentheses denotes the $l = 1^{st}$ region, in the UV region just before the first duality transformation in the cascade.

According to Seiberg duality, $\mathcal{N} = 1$ supersymmetric $SU(N)$ electric gauge theory with $N_f \in (3N/2, 3N)$ flavors, which becomes strongly coupled in the IR, flows to a nontrivial conformal IR fixed point where it joins a dual $SU(N_f - N)$ magnetic gauge theory with N_f flavors. Now if we consider the $SU(N+M)$ gauge theory and think of the other $SU(N)$ gauge group as a weakly gauged flavor symmetry, we have $\mathcal{N} = 1$ supersymmetric $SU(N+M)$ gauge theory with $2N$ flavors; its running is faster than the running of an $SU(N)$ gauge theory with $2(N+M)$ flavors. Therefore, the $SU(N+M)$ gauge theory would get strongly coupled faster in the IR and following Seiberg duality the appropriate description of the theory in this region is in terms of a weaker dual magnetic theory. The question of interest to us is the effective value of the anomalous dimension which dictates the flow. Although it is the $SU(N+M)$ factor that undergoes duality transformation in the first step of the cascade, the flow cannot be dictated simply by the fixed point of $SU(N+M)$ gauge theory with $2N$ flavors for two reasons. First, the $SU(N)$ group factor which gives a flavor symmetry to $SU(N+M)$ would itself get strongly coupled during part of the flow. Second, the running of the tree level coupling has a fixed point for anomalous dimension $\gamma_\eta = -1/2$. In fact, if we consider the two flows separately, the $SU(N+M)$ factor tends to make $\gamma < -1/2$ while the $SU(N)$ factor tends to make $\gamma > -1/2$, and the strengths are slightly different and that is where the corrections to the anomalous dimension will originate. Consider the non-trivial IR fixed point of the gauge couplings in the nonperturbative regime. The anomalous dimension $\gamma_1(1)$ that would follow from the fixed point of $SU(N+M)$ with $2N$ flavors ($\beta_1 = 0$) is

$$\gamma_1(1) = -\frac{1}{2} - \frac{3}{2}\frac{M}{N}. \tag{3.4}$$

Similarly, the anomalous dimension $\gamma_2(1)$ that would follow from the fixed point of $SU(N)$ with $2(N+M)$ flavors ($\beta_2 = 0$) is

$$\gamma_2(1) = -\frac{1}{2} + \frac{3}{2}\frac{M}{N+M}. \tag{3.5}$$

The duality transformation in the first step of the cascade occurs in the $SU(N+M)$ factor because it would run faster, when the two gauge factors are looked at separately. In terms of the anomalous dimensions, it receives more deviation from $-1/2$ than $SU(N)$ does. The effective value of anomalous mass dimension which guides the running of the physical couplings must lie in between. One assumption we make now is that the effective value of anomalous mass dimension γ depends on γ_1 and γ_2 linearly. Thus we write $\gamma = (a\gamma_1 + b\gamma_2)/(a+b)$, where a and b are constants such that $a + b = 1$. To fix a and b, we note that the gauge theory has the obvious symmetry $M \to -M$, $N \to N + M$, which to leading order in M/N is $M \to -M$ with N fixed. Clearly, γ should be even under this symmetry and so cannot depend on M/N at first order. The symmetry $M \to -M$, $N \to N + M$ implies $bM/(N+M) - aM/N = -bM/N + aM/(N+M)$ and thus $a = b = 1/2$. Thus γ is the average of γ_1 and γ_2,

$$\gamma(1) = -\frac{1}{2} - \frac{3}{4}\frac{M^2}{N(N+M)}. \tag{3.6}$$

Indeed, we shall see that this leads to results consistent with dual supergravity flux backreaction estimates [14, 15] where the leading order corrections to the anomalous dimension is expected to come at most at $\mathcal{O}(M^2/N^2)$. Moreover, it gives a picture with the duality cascade ending in a baryonic branch consistent with the discussions in [16, 18].

The resulting gauge theory after the first duality transformation is $SU(N-M)\times SU(N)$ and now the running of the $SU(N)$ factor coupling would be faster and it is its turn for a duality transformation as the renormalization group flows towards the IR. Now consider the gauge group $SU(N-(l-2)M) \times SU(N-(l-1)M)$ in the l^{th} region approaching the l^{th} step ($l \leq N/M$) in the duality cascade. For odd l, it is the gauge group whose parent is the gauge factor $SU(N+M)$ which undergoes a duality transformation, while for even l it is the one with the $SU(N)$ parent. It is convenient to use g_1 for the stronger gauge coupling which corresponds to the gauge group that undergoes a duality transformation and g_2 for the other from now on.

Again treating the weaker gauge symmetry as a flavor symmetry, we then have, for the flow from the $(l-1)^{th}$ to l^{th} duality transformations,

$$\gamma_1(l) = -\frac{1}{2} - C_{l-1}, \tag{3.7}$$

$$\gamma_2(l) = -\frac{1}{2} + C_{l-2}, \tag{3.8}$$

where

$$C_l \equiv \frac{3}{2}\frac{M}{N - lM}. \tag{3.9}$$

The anomalous dimension for the l^{th} region in the cascade then follows from the average of the two,

$$\gamma(l) = -\frac{1}{2} - \frac{1}{3}C_{l-2}C_{l-1} = -\frac{1}{2} - \frac{3M^2}{4(N+2M-lM)(N+M-lM)}. \tag{3.10}$$

Now we are ready to identify the anomalous dimensions,

$$\gamma_1(l) = \gamma_2(l) = \gamma_\eta(l) = \gamma(l). \tag{3.11}$$

With this common value of $\gamma(l)$ in (3.10), the running of the couplings in (3.1), (3.2) and (3.3) becomes

$$\mu\frac{dT_1(l)}{d\mu} = 3M - C_{l-2}M, \tag{3.12}$$

$$\mu\frac{dT_2(l)}{d\mu} = -3M - C_{l-1}M, \tag{3.13}$$

$$\mu\frac{d\eta(l)}{d\mu} = -\frac{2}{3}C_{l-2}C_{l-1}. \tag{3.14}$$

Thus, the effective running of the couplings makes g_1 get stronger while g_2 gets weaker as the theory flows to the IR. In some region during the flow the two couplings have about equal strength. The dimensionless tree level coupling η after the duality transformation goes like the inverse of that before the transformation and the corresponding β function changes sign. As the magnitude of the anomalous dimension across each step of the cascade changes because the changing matter content of the cascading theory, so do the coefficients in the logarithmic running of the couplings. We see from the β functions in (3.12) and (3.13) that the physical running of the gauge couplings has appropriate feature with $\mathcal{O}(M^2/N)$ corrections.

4. Supergravity side

4.1 Type IIB supergravity action

Type IIB supergravity is the effective low energy background of type IIB strings. In this section we want to review and write down a summary of the action and the general equations of motion of type IIB supergravity consisting the fields of interest to us here. The point here is to see the general relations among the fluxes and the metric. At the same time we will see some of the special cases in which the equations reduce and become simpler to deal with directly.

The pair of 16 component spinors of $\mathcal{N}=2$ supersymmetry in ten dimensions have the same chirality in IIB and the corresponding spinor representation can be written as $16+16$. The nonperturbative description of strings contains Dp-branes, which have p spatial and 1 time dimensions. In the IIB case, p is constrained to take on odd numbers. Our interest here is IIB backgrounds in the presence of D3- and D5-branes, and in particular on $AdS_5\times T^{1,1}$ background with the D5-branes wrapping the S^2 cycle of $T^{1,1}$ near the tip. The gauge theory dual to this supergravity theory is a nonconformal $\mathcal{N}=1$ supersymmetric $SU(N+M)\times SU(N)$ with bifundamental chiral superfields and a quartic tree level superpotential. The flow of the theory induces a backraction 2-form NS-NS potential. The relevant field content of type IIB supergravity are a dilaton Φ, RR 0-, 2- and 4-forms C_0, C_2 and C_4, and NS-NS 2-form B_2, with corresponding fluxes $F_1=dC_0$, $F_3=dC_2$, $F_5=dC_4$ and $H_3=dB_2$ [19]. We also use the same symbols for the partial derivatives of the fields, $F_1=\partial C_0$, $F_3=\partial C_2$, $F_5=\partial C_4$ and $H_3=\partial B_2$ as it should be clear from context which one is meant. We will use normalization in which the RR flux from a Dp-brane satisfies,

$$\int_{S^{8-p}}\star F_{p+2}=\frac{2\kappa^2\tau_p N}{g_s}\,,\qquad \tau_p=\frac{1}{\kappa}\sqrt{\pi}(4\pi^2\alpha')^{(3-p)/2}\,,\qquad \kappa=8\pi^{7/2}g_s\alpha'^2 \tag{4.1}$$

where F_{p+2} is $(p+2)$-form flux, τ_P is the Dp-brane tension, κ is the gravitational constant in ten dimensions, α' is the string scale (Regge slope), g_s is the string coupling and N is the number of Dp-branes.

For the stack of N regular and M fractional $D3$-branes we have

$$\frac{1}{(4\pi^2\alpha')^2}\int_{T^{1,1}}F_5=N\,,\qquad \frac{1}{4\pi^2\alpha'}\int_{S^3}F_3=M. \tag{4.2}$$

The bosonic part of type IIB classical effective supergravity action is then, in Einstein frame,

$$\begin{aligned}S_{10}=\frac{1}{2\kappa^2}\int\Big(&d^{10}x\sqrt{-G}\Big[R-\frac{1}{2}(\partial\Phi)^2-\frac{1}{2}g_s^2e^{2\Phi}F_1^2-\frac{1}{12}e^{-\Phi}H_3^2\\&-\frac{1}{12}g_s^2e^{\Phi}\tilde{F}_3^2-\frac{1}{4.5!}g_s^2\tilde{F}_5^2\Big]-\frac{1}{2}g_s^2C_4\wedge F_3\wedge H_3\Big),\end{aligned} \tag{4.3}$$

where

$$\tilde{F}_5\equiv F_5+B_2\wedge F_3\,,\qquad \tilde{F}_3\equiv F_3-C_0H_3\,, \tag{4.4}$$

G is the determinant of the metric in ten dimensions and R is the Ricci scalar. The 5-form flux is required to satisfy the self duality constraint

$$\star\tilde{F}_5=\tilde{F}_5 \tag{4.5}$$

and we write

$$\tilde{F}_5 = \mathcal{F}_5 + \star\mathcal{F}_5. \tag{4.6}$$

The corresponding equations of motion are

$$\begin{aligned}
R_{MN} = &\frac{1}{2}\partial_M\Phi\partial_N\Phi + \frac{1}{2}g_s^2 e^{2\Phi}\partial_M C_0\partial_N C_0 + \frac{1}{4}e^{-\Phi}(H_3)_{MOP}(H_3)_N^{OP} \\
&+\frac{1}{4}g_s^2 e^{\Phi}(\tilde{F}_3)_{MOP}(\tilde{F}_3)_N^{OP} + \frac{1}{96}g_s^2(\tilde{F}_5)_{MOPQR}(\tilde{F}_5)_N^{OPQR} \\
&-G_{MN}\Big(\frac{1}{48}e^{-\Phi}H_3^2 + \frac{1}{48}g_s^2 e^{\Phi}\tilde{F}_3^2 + \frac{1}{960}g_s^2\tilde{F}_5^2\Big),
\end{aligned} \tag{4.7}$$

$$d\star d\Phi = g_s^2 e^{2\Phi}F_1\wedge\star F_1 - \frac{1}{2}e^{-\Phi}H_3\wedge\star H_3 + \frac{1}{2}g_s^2 e^{\Phi}\tilde{F}_3\wedge\star\tilde{F}_3, \tag{4.8}$$

$$d\star(e^{2\Phi}F_1) = -e^{\Phi}H_3\wedge\star\tilde{F}_3, \tag{4.9}$$

$$d\star(e^{\Phi}\tilde{F}_3) = F_5\wedge H_3, \tag{4.10}$$

$$d\star(e^{-\Phi}H_3 - g_s^2 C_0 e^{\Phi}\tilde{F}_3) = -g_s^2 F_5\wedge F_3. \tag{4.11}$$

$$d\tilde{F}_5 = H_3\wedge F_3. \tag{4.12}$$

The uppercase indices $M, N, \dots$ above are for the ten dimensional spacetime coordinates and G_{MN} is the metric. Multiplying both sides of (4.7) by G^{MN} gives

$$R = \frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}g_s^2 e^{2\Phi}(\partial C_0)^2 + \frac{1}{24}e^{-\Phi}H_3^2 + \frac{1}{24}g_s^2 e^{\Phi}\tilde{F}_3^2. \tag{4.13}$$

We note a few general features of the theory from the above equations of motion. From (4.9) we see that the H_3 and the F_3 fluxes are perpendicular when $C_0 = 0$, which is the case in KS and we will set $C_0 = 0$ in our analysis from now on unless when we explicitly state otherwise. From (4.8) we see that the dilaton would be constant for a precise matching of the H_3 and F_3 fluxes such that $e^{-\Phi}H_3\wedge\star H_3 = g_s^2 e^{\Phi}F_3\wedge\star F_3$. In the case when the dilaton is taken constant, the equations of motion simplify and the solution on a singular conifold was found by Klebanov and Tseytlin [15], with the singularity at tip of the conifold where the radius of AdS_5 (or $T^{1,1}$) vanishes. However, with the KS picture in terms of Seiberg duality cascade, confinement via gaugino condensation at the end of the cascade on the gauge theory side leads to a deformed conifold with the tip being S^3. In this case, the tip of the cone is smoothed out and cut off at some finite r whose size depends of the magnitude of the 't Hooft coupling in the confining gauge group, g_sM. Thus one needs a metric and flux ansatz which takes into account the interpolation between S^3 at the tip and the asymptotically $S^2\times S^3$ geometry at large r. With a metric ansatz, one computes the Ricci scalar, and then equate is to (4.13) to determine the geometry and obtain the KS solution [3]. A second special case is where there are no D3- or D5-branes and both R-R fluxes F_5 and F_3 vanish while the NS-NS flux H_3 is turned on by wrapping NS5-branes on S^2. Thus the equations are simplified. Now we cannot have a constant dilaton, since the right hand side of (4.8) is nonzero. The solution to this case with $\mathcal{N} = 1$ supersymmetry was obtained by Maldacena and Nunez (MN) [20]. The KS solution on the deformed conifold with F_5 and F_3 fluxes turned on and the MN solution on the resolved conifold with the H_3 flux turned on are two well-known regular solutions on type IIB background with $\mathcal{N} = 1$ supersymmetry. A natural question was whether there existed a flow between them. The possibility for this was analyzed and a metric and a flux ansatz for it given by Papadopoulos and Tseytlin [21]. This issue was further investigated by Gubser, Herzog and Klebanov [16] who found a leading order perturbative solution around the KS solution. Butti, Grana, Minasian, Petrini, and Zaffaroni used $SU(3)$ structures to find a one parameter set of solutions which flow in a direction from KS to MN [18].

4.2 Mapping gauge coupling running to supergravity flow

In this section we want to apply the gauge/gravity duality to map the renormalization group flow of the gauge couplings to the running of the dilaton and the backreaction NS-NS 2-from potential.

The stack of M $D5$-branes wrapping S^2 of the $AdS_5 \times T^{1,1}$ background creates 3-form flux through S^3 which induces a backreaction 2-form potential B_2 in the S^2 cycle. The sum of the two gauge coupling coefficients $T_+ \equiv T_1 + T_2$, which can be taken as the effective gauge coupling, is related to the effective string coupling containing the dilaton in the dual gravity theory. The difference between the two gauge coupling coefficients $T_- \equiv T_1 - T_2$ is nonzero because the ranks of the two gauge groups are different and it describes the nonconformal nature of the theory. Indeed, T_- must dictate the flow in both the gauge and the gravity theories. We note that the supergravity equation of motion in the presence of nonzero R-R F_3 and F_5 fluxes from the $D3$- and $D5$- branes could have a consistent set of solutions only if the NS-NS 2-form potential B_2 is nonzero. Indeed, the two parameters T_+ and T_- on the gauge theory side are mapped to the effective string coupling and the 2-form potential B_2 through the relations [14, 22],

$$T_+ = T_1 + T_2 = \frac{2\pi}{g_s e^{\Phi}}, \tag{4.14}$$

$$T_- = T_1 - T_2 = \frac{2\pi}{g_s e^{\Phi}} \Big((b_2 + 1) \bmod 2 \Big), \tag{4.15}$$

where

$$b_2 \equiv \frac{1}{2\pi^2 \alpha'} \int_{S^2} B_2 \tag{4.16}$$

Note that we have defined b_2 such that the two gauge couplings are equal when $b_2 + 1 = 0$ with $b_2 = 0$ at the edge of the throat. In the KS solution the gauge couplings reach either infinity or zero when a Seiberg duality transformation occurs.

Here, with the corrections included, we see from (4.15) that the couplings get either large or small but never exactly infinity or zero.

Now for the flow from the $(l-1)^{th}$ to the l^{th} cascade steps we have

$$\frac{d}{d\log(\Lambda/\Lambda_c)}T_+ = -\Big(C_{l-2}+C_{l-1}\Big)M, \tag{4.17}$$

$$\frac{d}{d\log(\Lambda/\Lambda_c)}T_- = \Big(6+C_{l-1}-C_{l-2}\Big)M, \tag{4.18}$$

where Λ is the scale of the gauge theory and Λ_c is the cutoff. The scale of the gauge theory is mapped to the radial coordinate r of AdS_5 in the dual gravity theory,

$$\Lambda \sim r, \tag{4.19}$$

We note that the magnitude of the correction to the running of T_- increases with increasing l, since $C_{l-1} > C_{l-2}$. This increase in the coefficient of the logarithmic running of T_- will lead to a change of the slope in the logarithmic running of the B_2 potential which results in a step in the 3-form NS-NS flux H_3 and in the warp factor at a duality transformation. Applying the derivative with respect to $\ln(r/r_0)$ on (4.14) and (4.15), we obtain

$$\frac{d}{d\log{(r/r_0)}}\,e^{-\Phi} = -S_l, \tag{4.20}$$

$$\frac{d}{d\log{(r/r_0)}}\Big(e^{-\Phi}(b_2+1)\Big) = D_l, \tag{4.21}$$

where r_0 is the AdS_5 radius at the edge of the throat, (b_2+1) is defined mod 2 and we have introduced two sets of constants,

$$S_l \equiv (C_{l-1}+C_{l-2})\frac{g_s M}{2\pi}, \quad D_l \equiv (6+C_{l-1}-C_{l-2})\frac{g_s M}{2\pi} \tag{4.22}$$

Suppose the l^{th} duality transformation takes place at $r=r_l$. First we can solve (4.20) for $\Phi(r)$ in the range $r_l \le r \le r_{l-1}$ between the $(l-1)^{th}$ and l^{th} duality transformation locations imposing that the boundary condition $\Phi(r_0)=\Phi_0$ at $r=r_0$,

$$e^{-\Phi(r)} = e^{-\Phi_0} - \sum_{k=1}^{l-1} S_k \ln(r_k/r_{k-1}) - S_l \ln(r/r_{l-1}), \tag{4.23}$$

and $\Lambda/\Lambda_c = r/r_0$. The values of r_l can be computed using the change in magnitude of b_2, but the values in KS are good approximation for the UV region,

$$r_l \sim r_0 \exp\left(-\frac{2l\pi}{3g_s M}\right). \tag{4.24}$$

The leading term in the variation of $e^{-\Phi}$ comes at $\mathcal{O}(M^2/N)$ consistent with flux backreaction expectations [14].

Our interest is first to show how kinks appear in the running of the dilaton and the B_2 potential which lead to steps in the H_3 flux and in the warp factor. The expressions we present are only approximate and good for the large r region. We will not attempt to find the full supersymmetric solution on the supergravity side on the deformed/resolved conifold here. We seek a UV approximate expression for $b_2(r)$ with $b_2(r_0) = 0$ at the edge of the throat. This corresponds to the case in which the gauge coupling g_1 just starts getting stronger while g_2 starts getting weaker as the theory starts flowing down from the edge of the throat. We then obtain from (4.21) for r in the range $r_{l+1} \leq r \leq r_l$,

$$e^{-\Phi(r)} b_2(r) \sim e^{-\Phi(r_{l-1})} b_2(r_{l-1}) + D_l \ln(r/r_{l-1}). \tag{4.25}$$

The l^{th} duality transformation will occur at $r = r_l$ such that $b_2(r_l) = -2l$. With this and expressing $\ln(r/r_{l-1})$ in terms of Φ, (4.25) becomes

$$e^{-\Phi(r)} b_2(r) \sim -2(l-1)e^{-\Phi(r_{l-1})} + \frac{D_l}{S_l}\left(e^{-\Phi_0} - \sum_{k=1}^{l-1} S_k \ln(r_k/r_{k-1}) - e^{-\Phi(r)}\right). \tag{4.26}$$

The 2-form potential for the same range of r then follows from (4.16) and (4.25),

$$B_2(r) \sim b_2(r)\frac{\pi\alpha'\omega_2}{2}, \tag{4.27}$$

where w_2 denotes the S^2 cycle in $T^{1,1} = S^2 \times S^3$. The corresponding approximate 3-form NS-NS flux $H_3 = dB_2$ is

$$e^{-\Phi(r)} H_3(r) \sim \Big(D_l + S_l b_2(r)\Big)\frac{\pi\alpha' dr \wedge \omega_2}{2r}, \tag{4.28}$$

which has steps at Seiberg duality transformation locations. Each time r decreases past r_l, b_2 drops by 2, as implied by Seiberg duality and (4.15). As r decreases and past r_l, $b_2/2$ drops by one unit. Note also that the steps in the flux gives steps in the metric as the two are related by equations such as (4.13).

5. The warped deformed/resolved conifold

The deformation of the conifold via gaugino condensation in the gauge theory in the KS throat is related to a geometric transition in the gravity theory where the $S^2 \subset T^{1,1}$ cycle shrinks to zero size, the M number of $D5$ branes wrapping S^2 disappear and are replaced by flux through $S^3 \subset T^{1,1}$. Thus the tip of the deformed conifold is S^3. The metric which describes the deformed conifold thus involves an interpolation between $T^{1,1}$ at large r and S^3 at the tip of the conifold. However, it was later discussed that a flow to a baryonic branch with a quantum deformed moduli space of $SU(2M) \times SU(M)$, in the case where N is an integral multiple of M, might be the preferred route of the flow [16, 18]. In the MN case, $\mathcal{N}=1$ supersymmetric gauge theory was obtained by wrapping NS5-branes on S^2. A metric and flux ansatz which could give an interpolating solution between KS and MN was put forward in [21]. A leading order perturbative expansion around the KS solution was found in [16]. Later, $SU(3)$ structures were used to find a one parameter set of solutions which flow in a direction from KS to MN [18]. We used Einstein frame in previous sections. In this and the remaining sections, we will be using the string frame. The metrics in the two frames are related by $G_{MN}(\text{string}) = e^{\Phi/2} G_{MN}(\text{Einstein})$.

5.1 SU(3) structures

In this section we will briefly review the basic ideas in applying $SU(3)$ structures to study supergravity backgrounds with torsions. The study of supersymmetry conditions for supergravity backgrounds with torsion was initiated by Strominger [23]. See [24, 25, 26, 18] for details on applying group structures to supergravity.

Consider a compactification of type IIB strings on $R^{(1,3)} \times Y$, where Y is a compact six dimensional manifold. The Clifford algebra in ten dimensions is described by ten 32×32 gamma matrices. Let us denote these gamma matrices by Γ^M, where the uppercase letters $M, N, \cdots$ run over $0, 1, \cdots, 9$. The gamma matrices satisfy $\{\Gamma^M, \Gamma^N\} = 2G^{MN}$, where G_{MN} is the metric. The generators of the Lorentz group $Spin(1,9)$ on $R^{(1,3)} \times Y$ can be constructed as commutators of the gamma matrices. The spinor representation in 10-d is given by $\Gamma_{(10)} = \Gamma^0\Gamma^1 \cdots \Gamma^9$. The Lorentz algebra decomposes to $Spin(1,3) \times Spin(6)$ on $R^{(1,3)} \times Y$. We can write $\Gamma_{(10)} = \Gamma_{(4)}\Gamma_{(6)}$, where $\Gamma_{(4)} = -i\Gamma^0 \cdots \Gamma^3$ and $\Gamma_{(6)} = i\Gamma^4 \cdots \Gamma^9$ denote the spinor representations on $R^{(1,3)}$ and on Y respectively. There are two spinors of the same chirality in IIB which decompose under $Spin(1,3) \times Spin(6)$ as $\epsilon^1 = \zeta_+ \eta^1_+ + \zeta_- \eta^1_-$ and $\epsilon^2 = \zeta_+ \eta^2_+ + \zeta_- \eta^2_-$, where ζ_+ is the spinor on $R^{(1,3)}$, η^i are the spinors on Y, $\zeta_- = {\zeta_+}^*$, and $\eta^i_- = {\eta^i_+}^*$. The number of supersymmetries in 4-d depends on the structure group on Y. A generic Y with structure group $SO(6) \sim SU(4)$ has no globally defined covariantly constant spinor and gives no supersymmetry. The spinor representation of $SO(6)$ corresponds to the fundamental representation of $SU(4)$ which decomposes as $1 \oplus 3$ under $SU(3)$. Thus there is one globally defined $SU(3)$ singlet spinor on Y. In order to preserve some supersymmetry, Y needs to have a reduced structure group and to preserve $\mathcal{N}=1$ supersymmetry the structure group on Y has to be reduced at least to $SU(3)$. In that case, if we denote the one $SU(3)$ singlet spinor mentioned above by η_+, the two spinors $\eta^{1,2}_+$ are complex proportional and are related to the invariant spinor in terms of two complex functions α and β which can be expressed as $\eta^1_+ = \frac{1}{2}(\alpha + \beta)\eta_+$ and $\eta^2_+ = \frac{1}{2i}(\alpha - \beta)\eta_+$. If Y is a Calabi-Yau threefold, the globally invariant spinor would also be covariantly constant and depend trivially on the tangent frame bundle on Y and these two spinors give $\mathcal{N}=2$ supersymmetry in four dimensions.

However, when fluxes are turned on, the geometry backreacts and develops torsion and Y could in general become non-Ricci-flat and non-Kahler. When the extra space is compactified on a generalized Calabi-Yau with $SU(3)$ structures, the fluxes from the N regular and M fractional D3-branes give rise to torsions which fall in various representations of $SU(3)$. In the presence of fluxes, the spinor η^+ is not covariantly constant with respect to the Levi-Civita connection but would be so with a connection which includes torsion. The components of the torsion fall into the $SU(3)$ representations $(3+\bar{3})\otimes(3+\bar{3}+1) = (8+8)\oplus(6+\bar{6})\oplus(3+\bar{3})\oplus(3+\bar{3})\oplus(1+1)$. On the other hand, there are two $SU(3)$ singlets on Y, one is a fundamental 2-form which describes the almost complex structure and the other is a globally non-vanishing holomorphic 3-form. Unlike the case of Calabi-Yau threefolds, these 2- and 3-forms are not closed now and the different components of the torsion come in dJ and $d\Omega$. dJ has 20 components and decomposes under $SU(3)$ as $(6+\bar{6})\oplus(3+\bar{3})+(1+1)$, and $d\Omega$ transforms as a 24 of $SU(4)$ and decomposes under $SU(3)$ as $(8+8)\oplus(3+\bar{3})+(1+1)$. Similarly the fluxes can be decomposed into different components in representations of $SU(3)$. The different components of the torsion which fall in representations of $SU(3)$ need to vanish or get balanced by fluxes of the corresponding forms and representations in order to preserve $\mathcal{N}=1$ supersymmetry. This gives constraints on the relations among the parameters α and β, the fluxes and the metric.

Next we want to see the torsion components in the variations of the fundamental 2-form and the holomorphic 3-form when Y has $SU(3)$ structures. Suppose we have parameterized the metric on Y as

$$ds_6^2 = \sum_{m=1}^{6} G_m^2, \tag{5.1}$$

where G_m are real differential 1-forms which are not closed here. Lower case indices $m, n, \cdots$ run over 1 to 6 here. Let us then define

$$Z_1 = G_1 + iG_2, \quad Z_2 = G_3 + iG_4, \quad Z_3 = G_5 + iG_6. \tag{5.2}$$

We can then write the fundamental 2-form J and the holomorphic 3-form Ω as

$$J = \frac{i}{2}\sum_{i=\bar{i}=1}^{3} Z_i \wedge \bar{Z}_{\bar{i}}, \tag{5.3}$$

$$\Omega = Z_1 \wedge Z_2 \wedge Z_3. \tag{5.4}$$

The i is for holomorphic indices which run over 1 to 3, and the $\bar{i}$ is for the corresponding anti-holomorphic indices. However, the Z_i's are not differentials of complex coordinates and we will need to impose constraints in order to make Y a complex manifold. Note that J transforms as $(1,1)$ and Ω transforms as $(3,0)$. The complex and

Kahler structures on Y are determined by the properties in the variations of J and Ω. But it is easy to see that dJ has components with forms $(2,1)\oplus(1,2)\oplus(3,0)\oplus(0,3)$ in the Z_i's. Moreover, $d\Omega$ has components with $(3,1)\oplus(2,2)$ forms; it does not have a $(4,0)$, since a complex 4-form vanishes in three complex dimensions. When Y has $SU(3)$ structures, the components can further be broken down to representations of $SU(3)$. The $(3,0)\oplus(0,3)$ forms in dJ fall in the singlet representation, the $(1,2)\oplus(2,1)$ forms fall in the $(6\oplus 3)\oplus(\bar{6}\oplus\bar{3})$ representations. The $(3,1)$ form in $d\Omega$ falls in the $\bar{5}$ representation and the $(2,2)$ form falls in the $8\oplus 1$ representations. All in all,

$$dJ = -\frac{3}{2}\mathrm{Im}(W_1^{(1)}\bar{\Omega}) + (W_4^{(3)} + W_4^{(\bar{3})})\wedge J + (W_3^{(6)} + W_3^{(\bar{6})}), \tag{5.5}$$

$$d\Omega = W_1^{(1)} J^2 + W_2^{(8)}\wedge J + W_5^{(\bar{3})}\wedge\Omega, \tag{5.6}$$

where the W's denote components of the torsion. If Y is a Calabi-Yau manifold, then both J and Ω are closed, $dJ = 0$ and $d\Omega = 0$, and all torsion components vanish. Thus nonvanishing components of the torsion measure the departure of the manifold from being Calabi-Yau. The fluxes can also be decomposed as

$$H_3 = -\frac{3}{2}\mathrm{Im}(H_3^{(1)}\bar{\Omega}) + (H_3^{(3)} + H_3^{(\bar{3})})\wedge J + (H_3^{(6)} + H_3^{(\bar{6})}), \tag{5.7}$$

$$F_3 = -\frac{3}{2}\mathrm{Im}(F_3^{(1)}\bar{\Omega}) + (F_3^{(3)} + F_3^{(\bar{3})})\wedge J + (F_3^{(6)} + H_3^{(\bar{6})}). \tag{5.8}$$

If Y is to be a complex manifold, the $(3,0)$ and $(0,3)$ components of dJ and the $(2,2)$ components in $d\Omega$ must vanish which amount to demanding $W_1^{(1)} = 0$ and $W_2^{(8)} = 0$.

5.2 Equations of motion in the $6\oplus\bar{6}$ sector

The constraint on the relation between the fluxes and the torsions were found in [18]. It will be enough for our purpose here to focus only on the equations of motion in the $6\oplus\bar{6}$ sector. In particular the equations of motion for flux and torsion components in the $6\oplus\bar{6}$ representations are the following three complex equations, of which only two are independent,

$$(\alpha^2-\beta^2)W_3^{(6)} = 2\alpha\beta e^{\Phi}F_3^{(6)}, \tag{5.9}$$

$$(\alpha^2+\beta^2)W_3^{(6)} = -2i\alpha\beta *_6 H_3^{(6)}, \tag{5.10}$$

$$(\alpha^2-\beta^2)H_3^{(6)} = (\alpha^2+\beta^2)e^{\Phi} *_6 F_3^{(6)}. \tag{5.11}$$

A one parameter of numerical solution for the supersymmetry conditions using the ansatz in [21] was obtained in [18] for the case of α real and β imaginary, where the varying parameter arises from different possible values of the boundary value of dilaton at the very edge (or the very bottom) of the throat and the vacuum expectation value of the axionic scalar moduli field on the quantum deformed moduli space in the baryonic branch. We will see later that the supergravity side containing the corrections to the anomalous mass dimension from the gauge theory side does not fall into this solution. We will also see in the next section the implications of the corrections in terms of $SU(3)$ structures. For now, a simple way to see that the supergravity flow we have here is different is simply to note that the leading order correction to the running of the dilaton in (4.23), if we just consider the flow in the range $r_1 \leq r \leq r_0$ and define $\tilde{t} \equiv \ln(r/r_0)$, comes at $\mathcal{O}(\tilde{t})$, which is different from the flow found in [16] and [18], where the leading order correction to the running of the dilaton comes at $\mathcal{O}(\tilde{t}^2)$. It will be important to construct the full dual supergravity background and flow corresponding to the supersymmetric gauge theory containing corrections to the anomalous mass dimension.

6. Gravitational source for Seiberg duality transformations

Now we want to see that the locations where Seiberg duality transformations occur have a geometric obstruction with a jump in the relation between the two complex proportional spinors $\eta_+^{1,2}$ on the six dimensional manifold Y. Conversely, this geometric obstruction provides "special" locations on Y which source Seiberg duality transformations. First let us see the magnitudes of the "charges" (or the sizes of the steps) which come from the differences in the slopes in $e^{-\Phi}H_3$ given by (4.28) after and before a Seiberg duality transformation,

$$\begin{aligned}\Big(D_{l+1} - D_l\Big)\frac{\pi\alpha'}{2} &= (C_l + C_{l-2} - 2C_{l-1})\,\frac{g_s M\alpha'}{4} \\ &= \frac{3g_s M\alpha'}{4(K-l)(K-l+1)(K-l+2)}\end{aligned} \tag{6.1}$$

Note that, in the early stages of the duality cascade, the jump is of order $\mathcal{O}(1/K^3)$ and is quite small for large K. The magnitude increases as l increases, and the maximum value occurs at the last duality transformation where $(C_{K-1}+C_{K-3}-2C_{K-2}) = 1/2$ for $N = KM$. If we sum up the charges at each step, the magnitude of the total charge from the $K-1$ duality transformations is proportional to

$$\sum_{l=1}^{K-1} (C_l + C_{l-2} - 2C_{l-1}) = \frac{3}{4}\frac{(K-1)(K+2)}{K(K+1)} = \frac{3}{4}\Big(1 - \frac{2}{K(K+1)}\Big). \tag{6.2}$$

We see that in the limit $K \to \infty$, (6.2) goes to $3/4$. Of this the $(K-1)^{th}$ duality transformation takes $1/2$ and the $(K-2)^{th}$ duality transformation takes $1/8$. Most of the charge is located in the bottom region of the throat. Moreover, the charges confirm that the theory flows to a baryonic branch rather than to a confining branch and the duality cascade ends with the gauge group $SU(2M) \times SU(M)$, in the case where N is an integral multiple of M. That is because an additional duality transformation would need an infinite amount of charge, since (6.1) diverges for $l = K$.

This also agrees with the discussion and expectation in [16] and [18] of a flow along a baryonic branch.

The torsion and the flux components in the $6 \oplus \bar{6}$ sector for the ansatz given in [21] and used in [18] can be schematically written as $W_3^{(6)} = W_R + iW_I$, $H_3^{(6)} = H_R + iH_I$, and $F_3^{(6)} = F_R + iF_I$ and be split into two sets where the elements of a set have components in the same directions: $(W_R,\ F_I,\ *_6F_R,\ H_R,\ *_6H_I)$ and $(W_I,\ F_R,\ *_6F_I,\ H_I,\ *_6H_R)$ [27]. In other words, the elements in the first set, $W_R,\ F_I, \cdots$ have components along some directions such as $G_1 \wedge G_3 \wedge G_6$ while the elements in the second set, $W_I,\ F_R, \cdots$ have components along other directions such as $G_1 \wedge G_3 \wedge G_5$. If we let α and β have arbitrary phase θ between them and write $\beta = \tan \frac{w}{2} e^{i\theta} \alpha$, then the equations of motion in the $6 + \bar{6}$ sector, (5.9)-(5.11), give

$$\Big(1 - \tan^2 \frac{w}{2} \cos 2\theta \Big) W_R = -2 \tan \frac{w}{2} e^{\Phi} \sin \theta \, F_I, \tag{6.3}$$

$$\Big(1 - \tan^2 \frac{w}{2} \cos 2\theta \Big) W_I = 2 \tan \frac{w}{2} e^{\Phi} \sin \theta \, F_R, \tag{6.4}$$

$$\tan^2 \frac{w}{2} \sin 2\theta \, W_I = 2 \tan \frac{w}{2} e^{\Phi} \cos \theta \, F_R, \tag{6.5}$$

$$- \tan^2 \frac{w}{2} \sin 2\theta \, W_R = 2 \tan \frac{w}{2} e^{\Phi} \cos \theta \, F_I, \tag{6.6}$$

$$\Big(1 - \tan^2 \frac{w}{2} \cos 2\theta \Big) H_R = e^{\Phi} \Big(1 + \tan^2 \frac{w}{2} \cos 2\theta \Big) *_6 F_R, \tag{6.7}$$

$$\Big(1 - \tan^2 \frac{w}{2} \cos 2\theta \Big) H_I = e^{\Phi} \Big(1 + \tan^2 \frac{w}{2} \cos 2\theta \Big) *_6 F_I, \tag{6.8}$$

$$\tan^2 \frac{w}{2} \sin 2\theta \, H_I = -e^{\Phi} \tan^2 \frac{w}{2} \sin 2\theta \ *_6 F_I, \tag{6.9}$$

$$- \tan^2 \frac{w}{2} \sin 2\theta \, H_R = e^{\Phi} \tan^2 \frac{w}{2} \sin 2\theta \ *_6 F_R. \tag{6.10}$$

We see that the equations of motion are over-constrained and do not have solution for generic values of θ unless $\theta = \pm\frac{\pi}{2}$ (or for $\theta = 0, \pi$ with the fluxes and the torsions relabeled or related by S-duality).

In order to study the solutions which describe the effects of corrections to the anomalous mass dimension on the supergravity side, one needs a more general ansatz such as a complexification of the components F_R, F_I, H_R and H_I above and/or turning on F_1 flux in such a way that arbitrary phase between the two spinors could be accommodated. That is because the corrections change the imaginary self-duality condition in the 3-form combination $F_3^{(6)} - ie^{-\Phi} H_3^{(6)}$ in the KS solution in such a way that the supergravity flow does not occur at a fixed phase between the two spinors. The change in magnitude of corrections after a Seiberg duality transformation leads to a step in the function $\tan{(w/2)}\, e^{i\theta}$ which relates the two spinors. This results in a geometric obstruction. These special geometric locations and the charges on them provide a gravitational source for Seiberg duality transformations.

7. Discussions

The gauge/gravity duality implies that the nonperturbative dynamics of the gauge theory knows about the string background geometry. Here we have studied the implications of corrections to the anomalous mass dimension in the physical running of the couplings in the gauge theory to the dual gravity theory in the KS throat. We find that a more precise anomalous dimension on the gauge theory side reveals structures on the gravity side. The corrections make the dilaton and the potentials run with kinks and the fluxes and the metric have steps and the deviation from KS grows more and more as the duality cascade proceeds and the theory flows down to the bottom of the throat.

The magnitudes of the charges at the steps (or the sizes of the steps) are much smaller in the early stages of the cascade than in the last steps. The magnitudes of the charges confirm that the theory flows to a baryonic branch rather than to a confining branch and the duality cascade ends with the gauge group $SU(2M) \times SU(M)$, in the case where N is an integral multiple of M. That is because an additional duality transformation would require an infinite charge or a step with infinite size. This is consistent with what we would expect from the gauge theory side, since if we think of the $SU(M)$ in $SU(2M) \times SU(M)$ as a weakly gauged flavor symmetry, we have $SU(2M)$ with $2M$ flavors which falls far outside Seiberg's electric-magnetic duality window. Rather, it has a quantum deformed moduli space [28]. This also agrees with the discussion and expectation in [16] and [18] of a flow along a baryonic branch. Conversely, a duality cascade ending in the baryonic branch supports our premise that the anomalous dimension changes after every Seiberg duality transformation as the matter content of the theory changes with the $SU(2M) \times SU(M)$ not undergoing a duality transformation.

The steps also provide special locations with geometric obstructions which source Seiberg duality transformations. The steps we have discussed here are sharp because a Seiberg duality transformation occurs at a fixed $T^{1,1}$ base (or AdS_5 radius). The steps in the fluxes and the metric could be smoothed out if the charges at the steps get redistributed and a Seiberg duality transition takes place over some range of scales or AdS_5 radius. Conversely, the sharpness of the steps on the gravity side would provide a measure for the sharpness of Seiberg duality transitions on the gauge theory side.

It is believed that $NS5$-brane charges are located at the bottom of the KS throat. The $NS5$-branes wrap an S^2 of the S^3. In the absence of fluxes, they wrap a shrinking S^2 of the S^3 at, say, angular coordinate $\psi = 0$ and essentially vanish without a trace. If such $NS5$-brane can tunnel to the other pole of the S^3, i.e., $\psi = \pi$, it becomes M $D3$-branes [29]. Since there are K $NS5$-branes, together there are $N = KM$ $D3$ charges. Our picture suggests that these K $NS5$-branes are located at different positions, one at each $r = r_l$. Furthermore, they are at different locations of the S^3, that is, they wrap different shrinking S^2s of the S^3. As a consequence, H_3 is no longer orthogonal to F_3, which is the case with a non-zero R-R 1-form flux F_1.

The gauge/gravity duality is a powerful tool to probe both the gauge and the dual gravity theories from different directions. It seems that the best way to test/use gauge/gravity duality is an attempt to find the gravity dual to QCD. However, our theoretical control is best when we consider super Yang-Mills theories. Unfortunately, lattice gauge theory for super Yang-Mills theories is too rudimentary to be useful at the moment. Since Seiberg duality and gauge/gravity duality are strongly believed but not proven, it is quite amazing and useful that the whole notion of Seiberg and gauge/gravity dualities may be tested in cosmology. If our universe indeed resides at a bottom of such a warped deformed throat, the cosmological implication of this step-wise behavior of the metric on the brane inflationary scenario and the cosmic microwave background radiation can be very interesting.

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